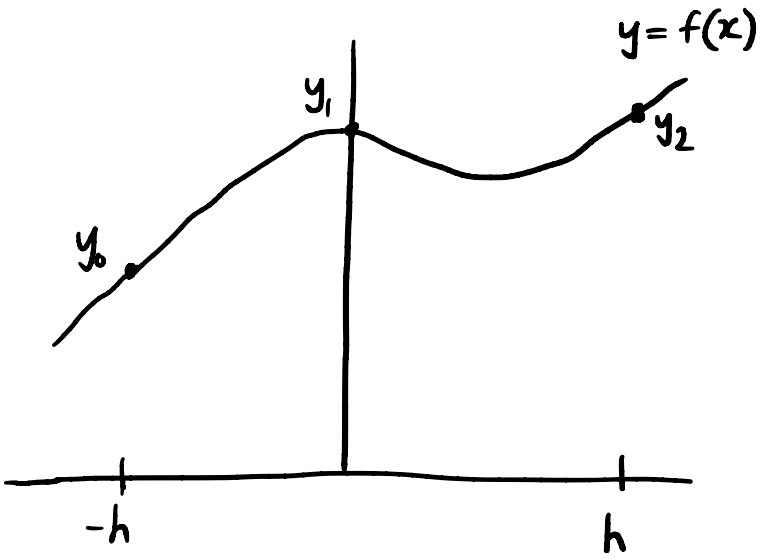
Derive Simpson's Rule



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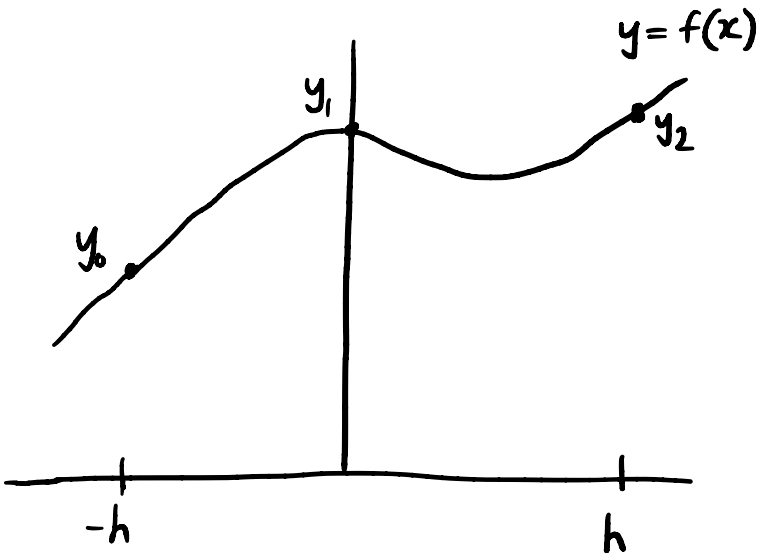

Quadratic $P(x) = Ax^2 + Bx + C$

$$\int_{-h}^{h} P(x)\,dx = \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx \Big|_{-h}^{h}$$

$$= \frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch - \left(-\frac{Ah^3}{3} + \frac{Bh^2}{2} - Ch\right)$$

$$= \frac{2}{3}Ah^3 + 2Ch$$

$$= \frac{h}{3}\left(2Ah^2 + 6C\right)$$

$$= \frac{h}{3}\left(2\left(-y_1 + \frac{y_2}{2} + \frac{y_0}{2}\right) + 6y_1\right)$$

$$= \frac{h}{3}\left(4y_1 + y_2 + y_0\right)$$

$$= \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

$$= \frac{\Delta x}{3}\left(y_0 + 4y_1 + y_2\right)$$

At $(0, y_1)$,

$y_1 = 0 + 0 + C \Rightarrow C = y_1$

$y_2 = Ah^2 + Bh + y_1$

$y_0 = Ah^2 - Bh + y_1$

$y_2 + y_0 = 2Ah^2 + 2y_1$

$\Rightarrow 2Ah^2 = -2y_1 + y_2 + y_0$

$Ah^2 = -y_1 + \frac{y_2}{2} + \frac{y_0}{2}$